

Statistical Tensile Strength for High Strain Rate of Aramid and UHMWPE Fibers

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Abstract: Dynamic tensile impact properties of aramid (Technora) and UHMWPE (DC851) fiber bundles were studied at two high strain rates by means of reflecting type Split Hopkinson Bar, and stress-strain curves of fiber yarns at different strain rates were obtained. Experimental results show that the initial elastic modulus, failure strength and unstable strain of aramid fiber yarns are strain rate insensitive, whereas the initial elastic modulus and unstable strain of UHMWPE fiber yarns are strain rate sensitive. A fiber-bundle statistical constitutive equation was used to describe the tensile behavior of aramid and UHMWPE fiber bundles at high strain rates. The good consistency between the simulated results and experimental data indicates that the modified double Weibull function can represent the tensile strength distribution of aramid and UHMWPE fibers and the method of extracting Weibull parameters from fiber bundles stress-strain data is valid.

Key words: aramid fiber; UHMWPE fiber; impact behaviour; statistical properties; statistical constitutive equation

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Technora, made from aramid fibers and somewhat similar to the commonly known Kevlar, and Ultra highly oriented high molecular weight polyethylene-UHMWPE fibers are all often used in flexible armour applications and hence subjected to high rates of loading. The utilization of their reinforced composites armour in certain ballistic applications is increasingly preferred over conventional rigid metal armour systems because of its superior strength-to-weight ratio and flexibility. To date, the design and development of such fabric armour systems have largely been approached empirically^[1-4]. The ballistic resistance of fiber-reinforced composite is essentially dependent on the mechanical behavior of the reinforcing component and the matrix. Fibers and fiber bundles carry the bulk of the applied load in a fiber-reinforced composite. Thus the understanding of the strain rate sensitivity of the fibers is very important for the use and design of the fiber reinforced composites. Nevertheless, for Technora and UHMWPE fibers and fiber bundles, few experimental results on

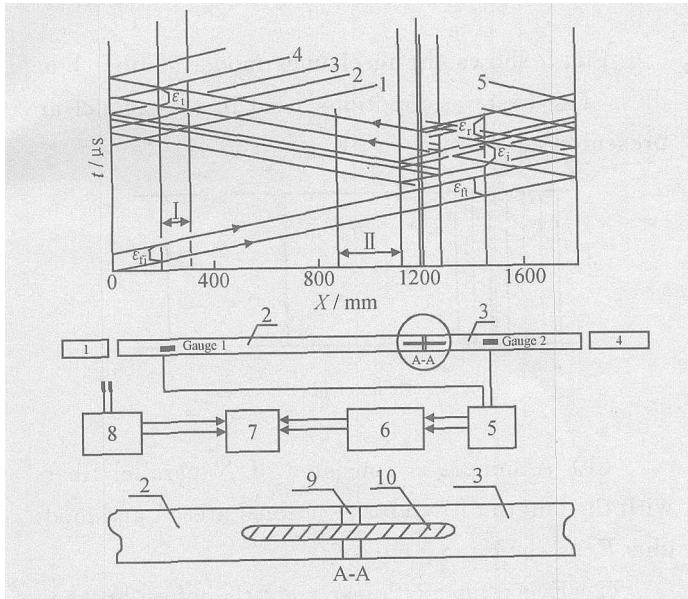
the high strain rate have been reported. It is well known that engineering fibers often exhibit a brittle fracture behavior and their tensile strength obey Weibull statistical distribution. Two experimental methods can be used to determine the strength statistical distribution parameters of fibers under quasi-static loading. One is the single fiber test and the other is the fiber bundle test^[5-11]. The fiber bundle testing method is increasingly used today. On the other hand, due to the technique difficulty, no suitable experimental technique is available to perform the dynamic single fiber tension tests. Therefore, the fiber bundle testing method is the only choice for determining the dynamic tensile strength distribution of fibers under impact loading conditions^[12-14]. In this paper, tensile impact tests of aramid (Technora) and UHMWPE (DC851) fiber bundles are carried out under two high strain rates by using a Split Hopkinson Tensile Bar (SHTB), and the experimental data are used to derive statistical constitutive equation of aramid and UHMPE fiber bundles.

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1 Experimental Apparatus and Tensile Impact Test

Tensile impact tests were conducted on the reflected indirect tensile impact apparatus from modified SHPB (Split Hopkinson Pressure Bar). A schematic of the setup and stress wave propagation $x-t$ scheme are presented in Fig. 1. Through the impact of the striker bar on the output bar, producing a stress wave via lining block transmits through the input bar. Such stress impulse travels down the input bar and is partially reflected at the input bar/specimen interface and partially transmitted to the specimen and the output bar. Measurements of the reflected and transmitted waves were made using strain gauges arranged on the respective bars of the system. The incident strain $\varepsilon_i(t)$, reflected strain $\varepsilon_r(t)$ and transmitted strain $\varepsilon_t(t)$ are recorded at strain gauges mounted on the input bar and output bars, respectively. According to the one-dimensional simple wave theory, the dynamic tensile stress $\sigma_s(t)$, dynamic tensile strain $\varepsilon(t)$ and strain rate $\dot{\varepsilon}(t)$ in the specimen can be obtained from the following:



1-striker bar; 2-output bar; 3-input bar; 4-absorbing bar;
5-super dynamic amplifier; 6-transient converter; 7-computer;
8-speedometer; 9-lining block; 10-specimen

Fig. 1 Tensile apparatus sketch and stress wave propagation $x-t$ scheme

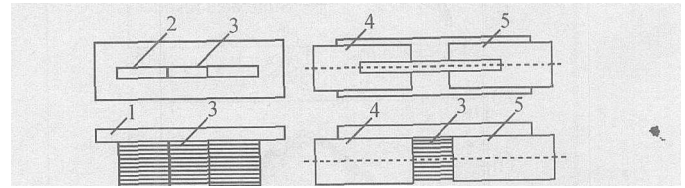
$$\sigma_s(t) = \frac{EA_0}{A_s} \varepsilon(t) \quad (1)$$

$$\varepsilon_s(t) = \frac{2C_0}{L_0} \int_0^t [\varepsilon(\tau) - \varepsilon(\tau)] d\tau \quad (2)$$

$$\dot{\varepsilon}_s(t) = \frac{2C_0}{L_0} [\varepsilon(t) - \varepsilon(t)] \quad (3)$$

where E and A_0 are the Young's modulus and the cross-sectional area of the input/output bar, C_0 is the longitudinal wave velocity of the bar, A_s and L_0 are the cross-sectional area and gage length of the specimen, respectively.

The material investigated in this paper are Technora (produced by Teijin in Japan) and DC851 (UHMWPE, produced by Ningbo Dachen in China) fiber bundle. A schematic diagram of the fiber bundle specimen and its connection with the input/output bars is shown in Fig. 2. The two lining blocks are glued to the supplement plate. The fiber bundles are wound continuously onto each side of the lining blocks in parallel and uniform windings. Then the lining blocks with the bundles are bonded into the slots of the input/output bars using high shear strength adhesive. The supplement plate is removed from the lining blocks before the tensile impact test. The specimen gage length is 8mm.



1-supplement plate; 2-lining block; 3-fiber bundles;
4-output bar; 5-input bar

Fig. 2 Fiber bundle specimen and its connection with input/output bars

The tensile impact tests were conducted at two high strain rates, 1700/s and 2500/s for Technora and 2200/s and 2500/s for UHMWPE fiber bundles. Fig. 3 and Fig. 4 show the integral tensile stress-strain curves of the two fiber bundles, respectively. The tensile properties of initial Young's modulus E , the maximum value σ_b (failure stress) and unstable strain ε_b (corresponding to the failure stress) for Technora and UHMWPE fiber bundles at two strain rates are listed in Table 1. It is seen that the initial Young's modulus, failure stress and unstable strain of Technora fiber does not apparently with the change of strain rate, so Technora fiber can be

regarded as high strain rate insensitive material. For UHMWPE (DC 851) fiber, the initial Young's modulus increases apparently with increasing strain rate, whereas the unstable strain decreases apparently with the strain rate increases, and the failure stress increases slightly, so the initial Young's modulus and unstable strain of UHMWPE (DC 851) fiber are high strain rate sensitive material. That's because the high-strength and high-modulus polyethylene fiber possesses characteristic with a high degree of crystal and orientation, most molecules participate in straight chain crystal, and all the straight chain crystal molecules participate in high rate impact tensile, thus the initial elastic modulus and unstable strain of high-strength and high-modulus polyethylene fiber yarns are strain rate sensitive on macroscopic.

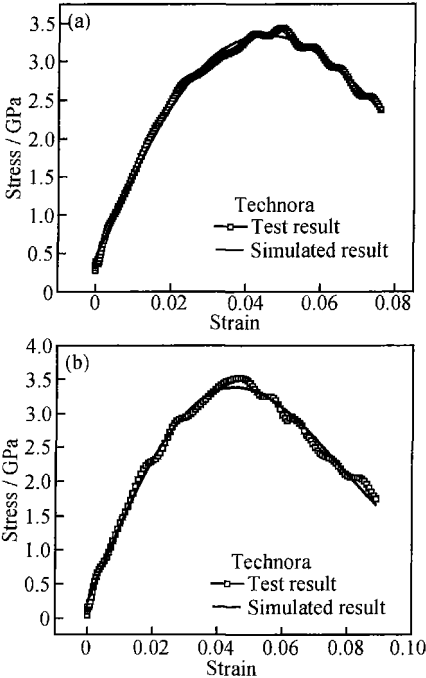


Fig. 3 Stress-strain curves of aramid (Technora) fiber bundles at two high strain rates
(a) strain rate= 1700/s; (b) strain rate= 2500/s

Table 1 Mechanical properties of Technora and DC851 fiber bundles

		E / GPa	σ_b / GPa	$\epsilon_b / \%$
Technora	1700/s	112	3.35	4.49
	2500/s	114	3.50	4.45
DC851	2200/s	56	2.36	10.85
	2500/s	103	2.47	7.11

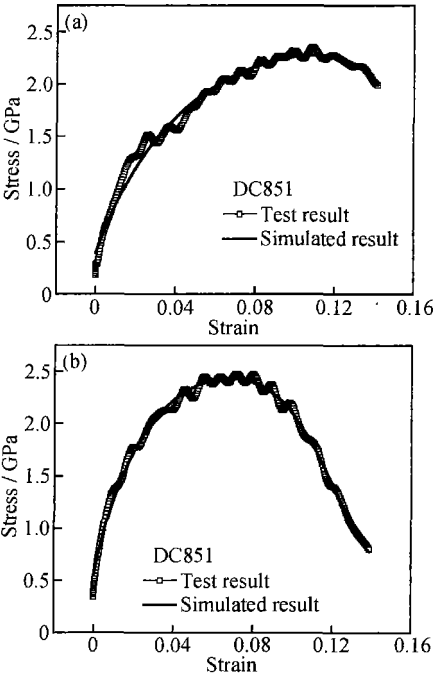


Fig. 4 Stress-strain curves of UHMWPE (DC851) fiber bundles at two high strain rates
(a) strain rate= 2200/s; (b) strain rate= 2500/s

2 Statistical Constitutive Model of Fiber Bundles

Fig. 5 shows the mechanics model of fiber bundles. The basic assumptions used in this model are presented as^[12- 16] :

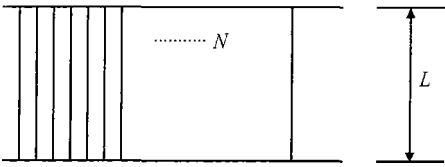


Fig. 5 Model of fiber bundles

- (1) A bundle is consisted of N parallel fibers with the length of L , cross-sectional area A and modulus E_f as shown in Figure 5.
- (2) Every fiber is linearly elastic before broken. There are no interactions among fibers. When n fibers are broken, the load will be evenly distributed to the rest of fibers.
- (3) The tensile strength of fibers follows a particular statistical distribution. Usually, the two- or four-parameter Weibull distribution expressed in Eq. (4) and Eq. (5) are adopted.

$$G(\varepsilon) = 1 - \exp\left[-\left[\frac{E\varepsilon}{\sigma_0}\right]^\beta\right] \quad (4)$$

(single Weibull distribution)

$$G(\varepsilon) = 1 - \exp\left[-\left[\frac{E\varepsilon}{\sigma_{01}}\right]^{\beta_1} - \left[\frac{E\varepsilon}{\sigma_{02}}\right]^{\beta_2}\right] \quad (5)$$

(double Weibull distribution)

where $G(\varepsilon)$ is failure probability of single fiber under an applied stress, E is Young's modules of fiber bundles, which can be experimentally evaluated from the stress-strain curve of fiber bundles, ε is strain of fiber bundles, σ_0 and β are the scale and shape parameters of Weibull distribution function, respectively. Thus the statistical constitutive model of fiber bundles can be expressed by:

$$\sigma = E\varepsilon \exp\left[-\left[\frac{E\varepsilon}{\sigma_0}\right]^\beta\right] \quad (6)$$

(single Weibull distribution)

$$\sigma = E\varepsilon \exp\left[-\left[\frac{E\varepsilon}{\sigma_{01}}\right]^{\beta_1} - \left[\frac{E\varepsilon}{\sigma_{02}}\right]^{\beta_2}\right] \quad (7)$$

(double Weibull distribution)

Based on the experimental stress-strain data, the experimental Weibull curve $\ln[-\ln(\frac{\sigma}{E\varepsilon})]$ against $\ln(E\varepsilon)$ can be obtained. Fig. 6 and Fig. 7 show the experimental Weibull curves at strain rates from 1700/s to 2500/s. It is indicated that the experimental Weibull graph is not a straight line and exhibits a nonlinear characteristic. Obviously, it is improper to use a straight line to extrapolate this experimental Weibull curve. The two- σ -parameter Weibull function is not appropriate for describing the strength statistical distribution of fibers, even though it has a simple form. Here a modified double Weibull distribution function is introduced to incorporate such nonlinear characteristic as shown in equation (8).

$$\sigma = \sigma_0 + E\varepsilon \exp\left[-\left[\frac{E\varepsilon}{\sigma_{01}}\right]^{\beta_1} - \left[\frac{E\varepsilon}{\sigma_{02}}\right]^{\beta_2}\right] \quad (8)$$

An algorithm for nonlinear least square estimation of parameters is used to simulate the experimental points and estimate the five Weibull parameters, σ_0 ; σ_{01} ; β_1 ; σ_{02} ; β_2 . The Weibull parameters of the Technora and UHMWPE fiber bundles at two strain rates are listed in Table 2. The comparison of the simulated stress-strain curves and Weibull curves with the experimental data is shown in Fig. 3 and Fig. 4, respectively. It can be seen that the simulated

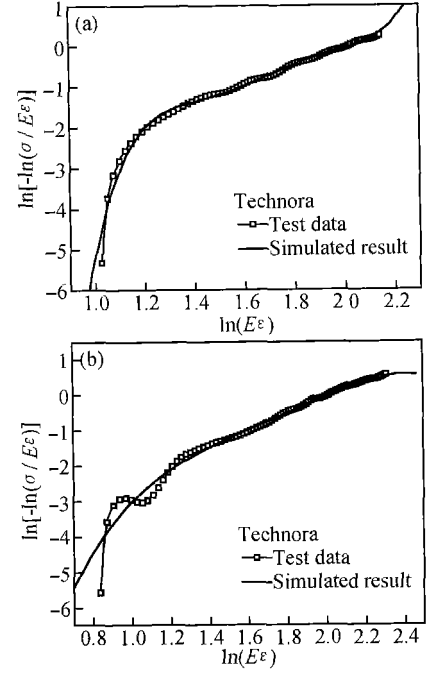


Fig. 6 Weibull distribution curves of Technora fiber bundles at two high strain rates
(a) strain rate= 1700/s; (b) strain rate= 2500/s

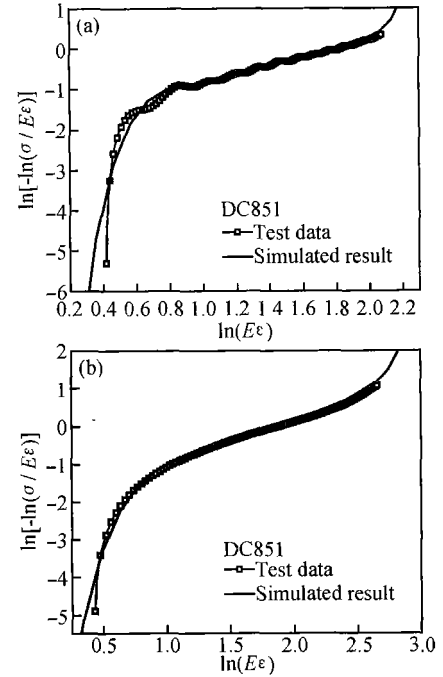


Fig. 7 Weibull distribution curves of DC851 fiber bundles at two high strain rates
(a) strain rate= 2200/s; (b) strain rate= 2500/s

results agree well with experimental data, which confirms that the modified double Weibull distribution function can represent the strength distribution of Technora and UHMWPE fiber bundles subjected to

high strain-rate loading. Using the parameters in Table 2 and Equation (8), we obtain statistical constitutive equations of fiber bundles at two different strain rates.

Table 2 Weibull parameters estimated from fiber bundle tests

		σ_0	α_1	β_1	σ_{02}	β_2
Technora	1700/s	0.43	12.14	6.79	7.05	1.80
	2500/s	0.50	7.87	3.09	6.85	3.08
DC851	2200/s	0.38	9.27	11.52	4.98	0.74
	2500/s	0.54	13.02	7.03	4.82	0.69

$$\sigma = 0.43 + 111.7\epsilon \exp\left[-\left[\frac{111.7\epsilon}{12.14}\right]^{6.79} - \left[\frac{111.7\epsilon}{7.05}\right]^{1.80}\right] \quad (9)$$

(Technora, $\epsilon = 1700/\text{s}$)

$$\sigma = 0.23 + 114.01\epsilon \exp\left[-\left[\frac{114.01\epsilon}{12.12}\right]^{2.05} - \left[\frac{114.01\epsilon}{8.99}\right]^{2.05}\right] \quad (10)$$

(Technora, $\epsilon = 2500/\text{s}$)

$$\sigma = 0.38 + 56.48\epsilon \exp\left[-\left[\frac{56.48\epsilon}{9.27}\right]^{11.52} - \left[\frac{56.48\epsilon}{4.98}\right]^{0.74}\right] \quad (11)$$

(DC851, $\epsilon = 2200/\text{s}$)

$$\sigma = 0.54 + 103.35\epsilon \exp\left[-\left[\frac{103.35\epsilon}{13.02}\right]^{7.03} - \left[\frac{103.35\epsilon}{4.82}\right]^{0.69}\right] \quad (12)$$

(DC851, $\epsilon = 2500/\text{s}$)

3 Conclusion

(1) Tensile impact tests on Technora and UHMWPE fiber bundles were carried out at two high strain rates. Experimental results show that Technora fiber can be regarded as high strain rate insensitive material, and the initial Young's modulus and unstable strain of UHMWPE (DC 851) fiber are high strain rate sensitive material.

(2) The fiber bundle constitutive equation can integrally describe the tensile behavior of Technora and UHMWPE fiber bundles at high strain rates. It is appropriate to use the modified double Weibull distribution function to characterize the strength distribution of Technora and UHMWPE fibers. The fi-

ber bundle testing method is valid to derive the dynamic Weibull parameters from fiber bundle stress-strain data.

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